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Inductance and resistance of bimetallic conductors

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INDUCTANCE AND RESISTANCE OF BIMETALLIC CONDUCTORS

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ABSTRACT

This paper presents a method of calculation of inductive reactance and resistance of a bimetallic conductor and compares the calculated values with the measured values.

A differential equation is written for the current distribution in a single strand of conductor. Using the boundary conditions of the problem, the current distribution is found. This, in turn, is used to find the expression for impedance of the conductor. The equation for impedance of a three strand bimetallic conductor is subsequently obtained by utilizing the above results and introducing values of permeability obtained experimentally.

A test procedure for measuring inductive reactance and resistance is discussed. A comparison of calculated versus measured values is made and the results presented in graphical form. Finally, the limitations of this approach are discussed.

INTRODUCTION

The purpose of this thesis is to discuss a method of calculating inductance and resistance of bimetallic conductors and to compare the calculations with measured values.

This thesis is divided into three parts. Namely,

1. the derivation of the equations and the calculation of transmission line impedance
2. the development of a test procedure and the determination of measured values of transmission line impedance
3. a comparison of the measured and calculated values

These tests were undertaken to determine the electrical and thermal properties of Bethalume Electrical Conductor[6]. This conductor consists of three or seven strands of wire, each strand consisting of a steel core coated with aluminum. The purpose of the steel core is to provide strength, while the aluminum coating provides good electrical conductivity and corrosion resistance.

In all, seven samples were tested, three samples of three strand conductor and four samples of the seven

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[6] All references are listed in the Bibliography

strand conductor.

The electrical tests fall into three categories:

1. tests for D.C. resistance
2. tests for A.C. resistance and inductive reactance at 60 cycles per second
3. tests for high frequency inductive reactance and charging current.

The calculation of the D.C. resistance and the high frequency inductive reactance and charging current is extensively covered in the literature and is not reviewed in this thesis. This thesis deals primarily with the calculation of the A.C. inductive reactance and resistance at 60 cycles per second. The calculations are based on the formulation now to be presented.

The inductive reactance and the resistance of the conductor as a function of current is calculated, assuming that the resistivity and permeability of the aluminum and the steel, and the dimensions of the conductor, the length, diameter, and spacing are given.

I. THE DERIVATION OF THE IMPEDANCE EQUATION

The general expression, with the assumptions listed below, for impedance which is derived here is applicable to a wire composed of any two materials carrying current of any frequency. This thesis is concerned primarily with aluminum coated steel wire carrying 60 cps current. Therefore, numerical results are calculated only for this particular type of conductor.

The assumptions made in this analysis are:

1. the spacing between the two wires is sufficiently large so that proximity effects may be neglected
2. the charging current is negligible in comparison to the total current
3. the current variation is sinusoidal.

Table 1 Symbols

Symbol	Explanation	Units
μ	permeability	henries/meter
ρ	resistivity	ohm-meters
r_2	radius of the wire	meters
r_1	radius of the steel core	meters
x	distance from the center of the wire to an arbitrary point within the wire	meters
B_x	flux density at a radius x	webers/meter^2
H_x	magnetic intensity at a radius x	ampere-turns/meter
E	induced voltage	volts
I_x	total current enclosed within a circle of radius x	amperes
i_x	current density at radius x	amperes/meter^2
f	frequency	cycles/second
$\omega = 2\pi f$	radian frequency	radians/second
ϕ	total flux linkages	webers
D	distance between the two conductors	meters
r	radius of the several strands spiralled together	meters
α	temperature coefficient	$1/^{\circ}F$
T	temperature	$^{\circ}F$

Solution

An equation is written for the current distribution in a single round wire. This expression is then generalized to give the current distribution in a round wire of two materials. After the current distribution is established, this information is used to find the expression for impedance of a single strand wire. This expression is then used to find the impedance in a multistrand case.

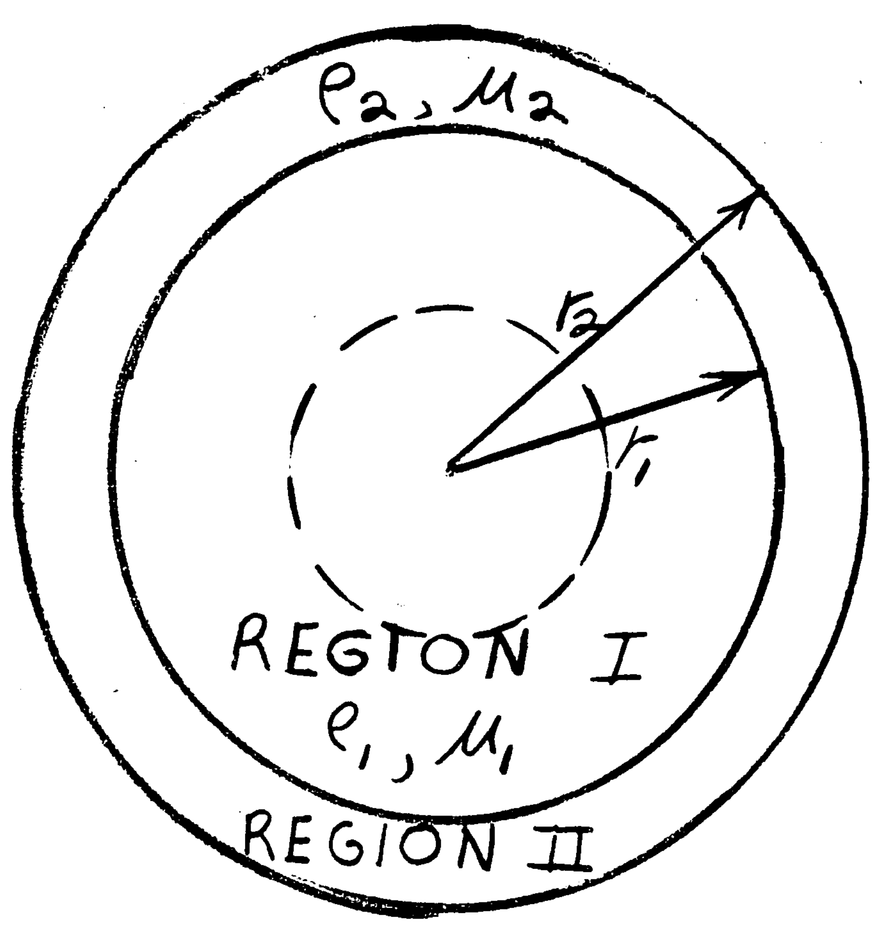

Figure 1
Cross sectional view
of bimetallic conductor

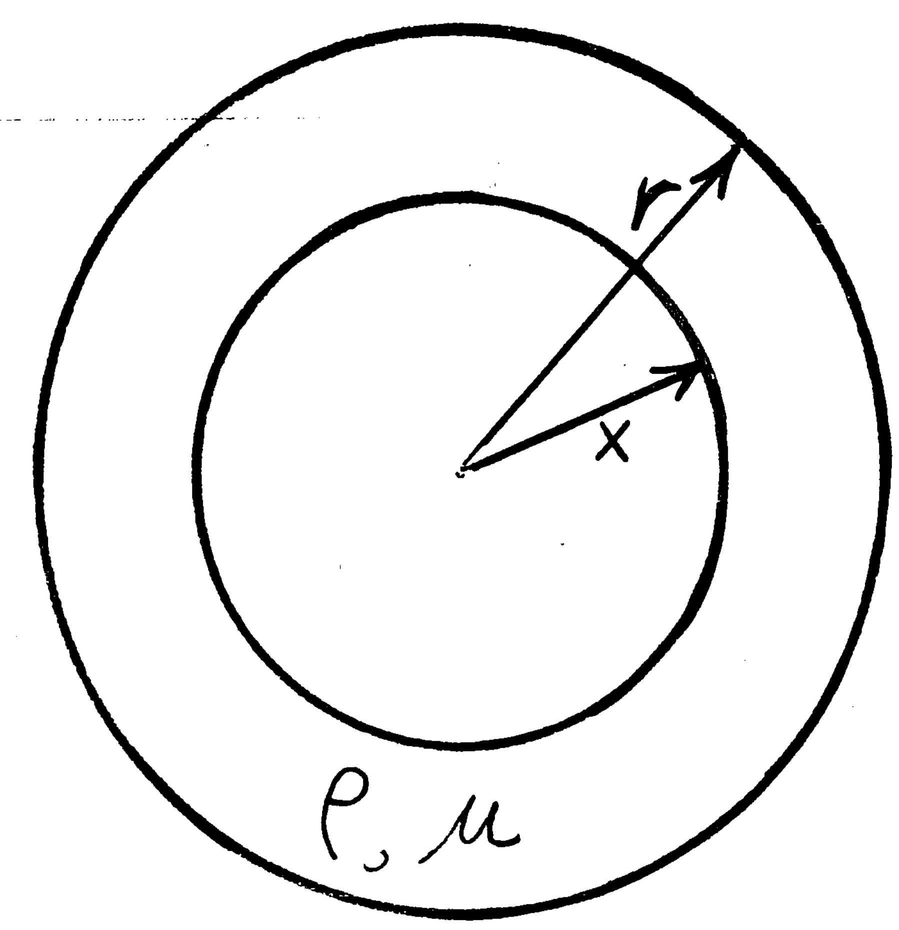

Figure 2
Cross sectional view
of cylindrical conductor

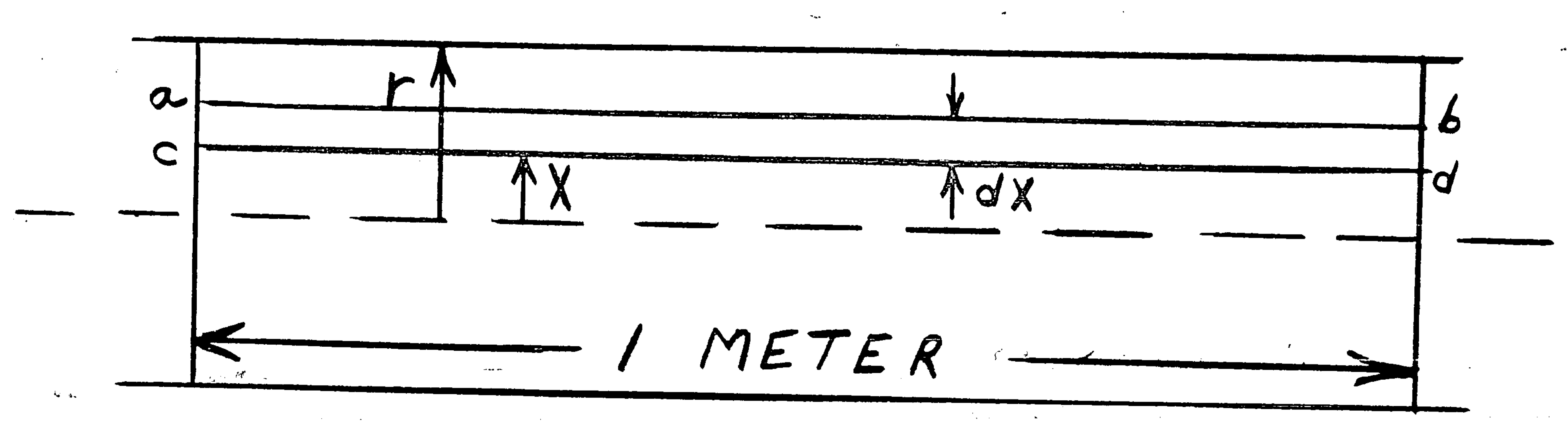

Figure 3
Diagram of a meter length of conductor

At any radius x in Fig. 2, the magnetizing force H_x is a constant since the flux lines are concentric. The total magnetizing force acting around a circle at radius x is

$$\oint H_x \cdot dl = 2\pi x H_x \qquad (1)$$

Ampere's Law states that the line integral of H around a single closed path is equal to the current enclosed. Thus

$$I_x = 2\pi X H_x \qquad (2)$$

or $$H_x = \frac{I_x}{2\pi X} \text{ ampere-turns per meter} \qquad (2\text{-a})$$

The total current flowing within the circle of radius x is

$$I_x = \int_0^x 2\pi x i_x dx \qquad (3)$$

Substituting Eq. 3 into Eq. 2-a

$$H_x = \frac{1}{2\pi x} \int_0^x 2\pi x i_x dx \qquad (4)$$

$$x H_x = \int_0^x x i_x dx \qquad (4\text{-a})$$

Differentiating

$$\frac{d}{dx}(x H_x) = \frac{d}{dx}\left[\int_0^x x i_x dx\right] \qquad (5)$$

$$H_x + x\frac{dH_x}{dx} = x i_x \qquad (5\text{-a})$$

or $$i_x = \frac{H_x}{x} + \frac{dH_x}{dx} \qquad (5\text{-b})$$

From Fig. 3, the total flux linking the rectangle abcda is found to be

$$\Phi = \mu H_x dx = B_x dx \qquad (6)$$

Faraday's Law states that the total emf induced in a closed circuit is directly proportional to the time rate of change of the total magnetic flux linking the circuit. Thus by differentiating Eq. 6

$$E = -\frac{d\phi}{dt} = -\mu\, dx\frac{dH_x}{dt} \tag{7}$$

The voltage drop between points a and b must be equal to the voltage drop between points c and d. By Eq. 7, the voltage drop due to the change in flux linkages is larger between points c and d than between points a and b by and amount $\mu\, dx\frac{dH_x}{dt}$. This difference must be accounted for by a greater resistive drop between points c and d. Equating the expression for the difference in voltage due to flux linkages to that for the difference in voltage due to resistive drop.

$$\mu\, dx\frac{dH_x}{dt} = \rho\, dx\frac{di_x}{dx} \tag{8}$$

Where $\rho\, dx\frac{di_x}{dx}$ is the resistance drop for a unit length of conductor.

Eq.8 may be rewritten as

$$\frac{dH_x}{dt} = \frac{\rho}{\mu}\,\frac{di_x}{dx} \tag{8-a}$$

Differentiating Eq. 5-b with respect to time

$$\frac{di_x}{dt} = \frac{1}{x}\,\frac{dH_x}{dt} + \frac{d^2H_x}{dt\,dx} \tag{9}$$

Subtituting the value of $\frac{dH_x}{dt}$ from Eq. 8-a into Eq. 9

$$\frac{di_x}{dt} = \frac{1}{x}\,\frac{\rho}{\mu}\,\frac{di_x}{dx} + \frac{d}{dx}\left[\frac{\rho}{\mu}\,\frac{di_x}{dx}\right] \tag{10}$$

$$\frac{di_x}{dt} = \frac{1}{x}\,\frac{\rho}{\mu}\,\frac{di_x}{dx} + \frac{\rho}{\mu}\,\frac{d^2 i_x}{dx^2}$$

or

$$\frac{d^2 i_x}{dx^2} + \frac{1}{x}\,\frac{di_x}{dx} = \frac{\mu}{\rho}\,\frac{di_x}{dt} \qquad (10\text{-a})$$

Since the current variation is assumed sinusoidal

$$i_x = i_{mx}\,\epsilon^{j\omega\tau}$$

Where i_{mx} is the maximum current density at radius x.

Therefore,

$$\frac{di_x}{dt} = j\omega\, i_{mx}\,\epsilon^{j\omega\tau} = j\omega\, i_x \qquad (11)$$

Substituting this into Eq. 10-a

$$\frac{d^2 i_x}{dx^2} + \frac{1}{x}\,\frac{di_x}{dx} - \frac{\mu}{\rho} j\omega i_x = 0 \qquad (12)$$

This is the fundamental equation for the current density in a conductor[7]. To solve this equation, first assume a solution of the form:

$$i_x = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \text{--------} \qquad (13)$$

and find $\frac{1}{x}\frac{di_x}{dx}$ and $\frac{d^2 i_x}{dx^2}$

$$\frac{1}{x}\,\frac{di_x}{dx} = \frac{a_1}{x} + 2a_2 + 3a_3 x + 4a_4 x^2 + \text{----} \qquad (13\text{-a})$$

$$\frac{d^2 i_x}{dx^2} = 2a_2 + 6a_3 x + 12a_4 x^2 + \text{----} \qquad (13\text{-b})$$

Substituting these values into Eq. 12

$$\frac{a_1}{x} + \left(4a_2 - \frac{\mu}{\rho} j\omega a_0\right) + \left(9a_3 - \frac{\mu}{\rho} j\omega a_1\right)x + \left(16a_4 - \frac{\mu}{\rho} j\omega a_2\right)x^2$$

$$+ \text{------} = 0 \qquad (14)$$

Equating coefficients of powers of x and taking a_0 as an arbitrary constant

$$a_0 = a_0$$

$$a_1 = 0$$

$$a_2 = \frac{\mu}{4\rho} j\omega \, a_0$$

$$a_3 = 0$$

$$a_4 = \frac{\mu}{16\rho} j\omega \, a_2 = -\frac{\mu^2\omega^2}{64\rho^2} a_0 \qquad (15)$$

Thus the expression for i_x is

$$i_x = a_0\left(1 + j\frac{\mu\omega}{4\rho}x^2 - \frac{\mu^2\omega^2}{64\rho^2}x^4 - j\frac{\mu^3\omega^3}{64\cdot 36\rho^3}x^6 + \text{----}\right) \qquad (16)$$

$$= a_0\, J_0\left(j^{3/2}\sqrt{\frac{\mu\omega}{\rho}}\,x\right) = a_0\, J_0(j^{3/2}mx) \qquad (17)$$

Where $J_0(j^{3/2}mx)$ is a Bessel Function of the first kind and of order zero.

Eq. 12 has a second solution of the form

$$i_x = b\, K_0(j^{1/2}mx) \qquad (18)$$

The complete solution of Eq. 12 is then

$$i_x = a_0\, J_0(j^{3/2}mx) + b\, K_0(j^{1/2}mx) \qquad (19)$$ 1b

Each region in Fig. 1 must satisfy Eq. 19, so that

$$i_{x1} = c_1\, J_0(j^{3/2}m_1x) + c_2\, K_0(j^{1/2}m_1x) \qquad (20)$$

$$i_{x2} = c_3\, J_0(j^{3/2}m_2x) + c_4\, K_0(j^{1/2}m_2x) \qquad (21)$$ 5

The constants c_1, c_2, c_3, and c_4 in Eqs. 20 and 21 are found by using the boundry conditions[3,4].
At $x = 0$, i_x remains finite but $K_0\,(j^{1/2}m_1x)$ becomes infinite. Therefore $c_2 = 0$

and $$i_{x1} = c_1\, J_0(j^{3/2}m_1x) \qquad (22)$$

but $J_0(0) = 1$

therefore $c_1 = i_0$, the current density at $x = 0$

and $i_{x1} = i_0\, J_0(j^{3/2} m_1 x)$ (22-a)

From Maxwell's Equations, it is known that the tangential component of H is continuous across a surface.

Thus at $x = r_1$ $\quad H_{x1} = H_{x2}$ (23)

Rewritting Eq. 8-a here

$$\frac{dH_x}{dt} = \frac{\rho}{\mu}\frac{di_x}{dx}$$

Therefore
$$\frac{\rho_1}{\mu_1}\frac{di_{x1}}{dx} = \frac{\rho_2}{\mu_2}\frac{di_{x2}}{dx} \tag{24}$$

Also at $x = r_1$ $\quad \rho_1 i_{x1} = \rho_2 i_{x2}$ (25)

Substituting the values of i_{x1} and i_{x2} from Eqs. 21 and 22-a into Eq. 25

$$\rho_2 c_3\, J_0(j^{3/2} m_2 r_1) + \rho_2 c_4\, K_0(j^{1/2} m_2 r_1) = \rho_1 i_0\, J_0(j^{3/2} m_1 r_1) \tag{26}$$

Substituting the same values of i_{x1} and i_{x2} into Eq. 24

$$\frac{\rho_1}{\mu_1}\frac{d}{dx}\left[i_0\, J_0(j^{3/2} m_1 x)\right]\Big|_{x=r_1} = \frac{\rho_2}{\mu_2}\frac{d}{dx}\left[c_3 J_0(j^{3/2} m_2 x) + c_4\, K_0(j^{1/2} m_2 x)\right]\Big|_{x=r_1}$$

$$-i_0 \frac{\rho_1}{\mu_1} j^{3/2} m_1\, J_1(j^{3/2} m_1 x)\Big|_{x=r_1} = \Big[-c_3 \frac{\rho_2}{\mu_2} j^{3/2} m_2\, J_1(j^{3/2} m_2 x) - c_4 \frac{\rho_2}{\mu_2} j^{1/2} m_2\, K_1(j^{1/2} m_2 x)\Big]\Big|_{x=r_1}$$

There results

$$c_3 \frac{\rho_2 m_2}{\mu_2} J_1(j^{3/2} m_2 r_1) - j\, \frac{c_4 \rho_2 m_2}{\mu_2} K_1(j^{1/2} m_2 r_1) = \frac{i_0 \rho_1 m_1}{\mu_1} J_1(j^{3/2} m_1 r_1) \tag{27}$$

Solving Eqs. 26 and 27 for c_3 and c_4

$$c_3 = \frac{\rho_1 m_1 \mu_2 i_0 J_1(j^{3/2} m_1 r_1) K_0(j^{1/2} m_2 r_1) + j \mu_1 m_2 \rho_1 i_0 K_1(j^{1/2} m_2 r_1) J_0(j^{3/2} m_1 r_1)}{\mu_1 \rho_2 m_2 \left[K_0(j^{1/2} m_2 r_1) J_1(j^{3/2} m_2 r_1) + j K_1(j^{1/2} m_2 r_1) J_0(j^{3/2} m_2 r_1)\right)} \tag{28}$$

$$c_4 = \frac{\rho_1\mu_1 m_2 i_0 J_0(j^{3/2}m_1r_1)J_1(j^{3/2}m_2r_1) - \rho_1\mu_2\, m_1 i_0 J_1(j^{3/2}m_1r_1)J_0(j^{3/2}m_2r_1)}{\mu_1\rho_2 m_2\left[K_0(j^{1/2}m_2r_1)\; J_1(j^{3/2}m_2r_1) + jK_1(j^{1/2}m_2r_1)J_0(j^{3/2}m_2r_1)\right]} \qquad (29)$$

The total current is

$$I = \int_0^{r_1} 2\pi x\; i_{x1}dx + \int_{r_1}^{r_2} 2\pi x i_{x2}dx$$

$$= 2\pi i_0\int_0^{r_1} x\; J_0(j^{3/2}m_1x)dx + 2\pi c_3\int_{r_1}^{r_2} x\; J_0(j^{3/2}m_2x)dx$$

$$+ 2\pi c_4\int_{r_1}^{r_2} x\; K_0(j^{1/2}m_2x) \qquad (30)$$

But

$$\int_a^b x\; J_0(j^{3/2}mx) = \left.\frac{x\; J_1(j^{3/2}mx)}{j^{3/2}m}\right|_a^b \qquad (31)$$

$$\int_a^b x\; K_0(j^{1/2}mx) = \left.\frac{-x\; K_1(j^{1/2}mx)}{j^{1/2}m}\right|_a^b \qquad (32)^{1b}$$

Therefore Eq. 30 becomes

$$I = \frac{2\pi i_0 r_1}{j^{3/2}m_1}\; J_1(j^{3/2}m_1r_1) + \frac{2\pi c_3\left[r_2\right.}{j^{3/2}m_2}\; J_1(j^{3/2}m_2r_2) - r_1\; J_1(j^{3/2}m_2r_1)\Big]$$

$$\frac{-2\pi c_4\left[r_2\right.}{j^{3/2}m_2}\; K_1(j^{1/2}m_2r_2) - r_1\; K_1(j^{1/2}m_2r_1)\Big] \qquad (33)$$

On the surface of the conductor there is no induced voltage due to the internal flux. The voltage drop along the surface of the wire minus the component of voltage due to external flux linkages is

$$e = \rho_2 i_2$$

$$= c_3\rho_2\; J_0(j^{3/2}m_2r_2) + c_4\rho_2\; K_0(j^{1/2}m_2r_2) \qquad (34)$$

The voltage drop per unit length of wire must be the same at any radius x in the conductor. Thus the surface voltage drop given in Eq. 34 is equal to that of the whole conductor. The impedance of the conductor is then

$$Z = \frac{e}{i} = \left[c_3 \rho_2 \; J_0(j^{3/2} m_2 r_2) + c_4 \rho_2 \; K_0(j^{1/2} m_2 r_2)\right] \div$$

$$2\pi \left\{ \frac{j^{3/2} i_0 r_1}{m_1} \; J_1(j^{3/2} m_1 r_1) + \frac{j^{3/2} c_3}{m_2}\left[r_2 \; J_1(j^{3/2} m_2 r_2) - r_1 \; J_1(j^{3/2} m_2 r_1)\right] \right.$$

$$\left. + \frac{c_4}{j^{3/2} m_2}\left[r_2 \; K_1(j^{1/2} m_2 r_2) - r_1 \; K_1(j^{1/2} m_2 r_1)\right] \right\}$$

$$+ 2 \cdot 10^{-7} \ln \frac{D}{r} \qquad \text{ohms per meter} \qquad (35)$$

Where the second term is added to account for the voltage drop due to the spacing component of the inductive reactance drop.

Eq. 35 is the general expression for the impedance of a single strand of bimetallic conductor.

II THE TEST PROCEDURE

Tests were made to find the quantities:

1. D.C. resistance
2. 60 cycle A.C. resistance as a function of current
3. 60 cycle A.C. inductive reactance as a function of current
4. Temperature rise of the wire, as a function of time, for given values of current
5. Inductive reactance, as a function of frequency, for frequencies to 200 kilocycles per second
6. Charging current, as a function of frequency, for frequencies to 200 kilocycles per second

The 60 cycle A.C. tests were made as follows. The test line was set up and connected as shown in Fig. 4. Measurements were made to determine 1. the complex impedance of the line at 68°F for various values of current, 2. the equilibrium-temperature complex impedance for various values of line current, and 3. the variation of the line temperature as a function of time after the A.C. source was applied.

The current to the line was controlled by using step- down transformers and variable resistors in series with the line. The current was measured by

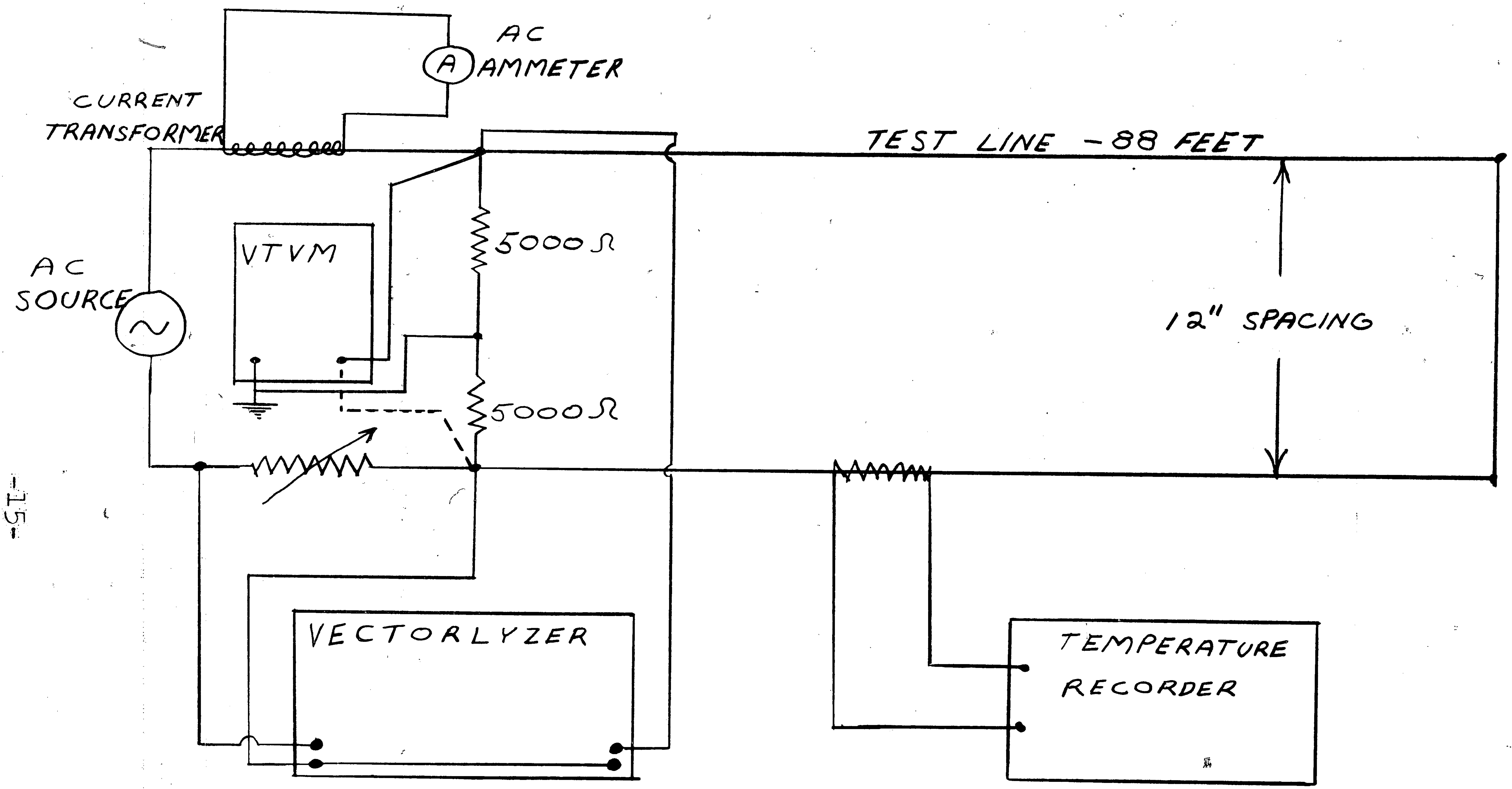

FIG. 4

TEST SETUP FOR 60 CYCLE A.C. MEASUREMENTS using a current transformer and an A.C. ammeter. A vacumn tube voltmeter was used as shown in Fig. 4 to measure the voltage. The phase angle of the transmission line impedance was measured using a Vectorlyzer. This instrument measures the phase angle between two voltages, in this case the voltage across the resistor and the transmission line. With one phase angle known, that of the resistor, the other is easily found. A temperature recorder was used to record the temperature as a function of time for the various current levels.

III PROPERTIES OF THE TEST SAMPLES

Due to the process by which the aluminum coating is applied to the wire, the thickness of the aluminum coating is relatively independent of the wire to which it is applied, except in the case of very small strands. A thickness of 0.0030 inches of aluminum is used, as the basis of calculation, for all samples except the 0.080 inch strand, for which a thickness of 0.0020 inches is used.

The same value of resistivity is used for all of of the Siemans-Martin, utility, and high-strength grades of steel. The resistivity of the extra high strength sample is assumed to be greater due to the additional working of the metal in its preparation.

The following values of resistivity are used.

$\rho_2 = \rho_{aluminum} = 2.83 \times 10^{-6}$ ohm- cm. at 20°C

$\rho_1 = \rho_{steel} = 25 \times 10^{-6}$ ohm- cm. at 20°C

$\rho_{extra\ high\ strength} = 35 \times 10^{-6}$ ohm- cm. at 20°C

A temperature coefficient $\alpha = 0.0031 \frac{1}{°F}$ is used. The calculated resistance is based on the values of resistivity and coating thickness given above. Both the measured and the calculated values given in Table II are for a temperature of 20°C.

Table II A Description of the Test Samples

Sample Number	Nominal Diameter	Strands	Grade	Diameter Strand	Area of Aluminum	D.C. Resistance (Cal.)	(Meas.)
1	1/4(Inches)	3	utility	0.121(In.)	0.0030 (In^2)	2.05	2.03 (Ohms/1000)
2	1/4	7	high strength	0.080	0.0032	2.04	2.12
3	3/8	3	utility	0.164	0.0046	1.18	1.17
4	3/8	3	extra high strength	0.164	0.0046	1.43	1.44
5	3/8	7	Siemans-Martin	0.121	0.0080	0.84	0.87
6	3/8	7	high strength	0.121	0.0080	0.84	0.87
7	1/2	7	high strength	0.164	0.0108	0.50	0.48

Assuming that the resistance and the internal component of the inductive reactance for a three strand wire is one-third the value of that of a single strand conductor, the value of the impedance given by Eq. 35 reduce to the following[2]:

Sample Number

1. $$Z = \cfrac{1}{\cfrac{8.17 \angle 44.8^\circ}{\sqrt{\mu'}\, J_B(j^{1/2} 0.063 \sqrt{\mu'})} + 0.164 \angle -3.3^\circ} + 0.108 \angle 90^\circ$$

ohms/1000'

3. $$Z = \cfrac{1}{\cfrac{11.7 \angle 45.0^\circ}{\sqrt{\mu'}\, J_B(j^{1/2} 0.087 \sqrt{\mu'})} + 0.262 \angle -1.3^\circ} + 0.0985 \angle 90^\circ$$

ohms/1000'

4. $$Z = \cfrac{1}{\cfrac{8.35 \angle 43.7^\circ}{\sqrt{\mu'}\, J_B(j^{1/2} 0.074 \sqrt{\mu'})} + 0.332 \angle 0^\circ} + 0.0990 \angle 90$$

ohms/1000'

$$\text{Where } J_B(j^{1/2} K\sqrt{\mu'}) = \text{conjugate } \frac{J_0(j^{1/2} K\sqrt{\mu'})}{J_1(j^{1/2} K\sqrt{\mu'})}$$

$$j \text{ conjugate } \frac{J_0(j^{1/2} K\sqrt{\mu'})}{J_1(j^{1/2} K\sqrt{\mu'})}$$

and μ' is a constant to be determined experimentally.

No attempt is made to solve for the exact current distribution in the case of the three spiralled strands. The expression for impedance of a single strand conductor is divided by three to account for the three parallel paths, and μ' is introduced to account for any variation from this value.

The quantity μ' is dependent on the current density in the test sample. Values of μ' were used which when substituted into the expression for the impedance of sample number one yielded satisfactory results. These same values of μ' were used for the other two samples. Fig. 5 shows the values of μ' used, as a function of current density.

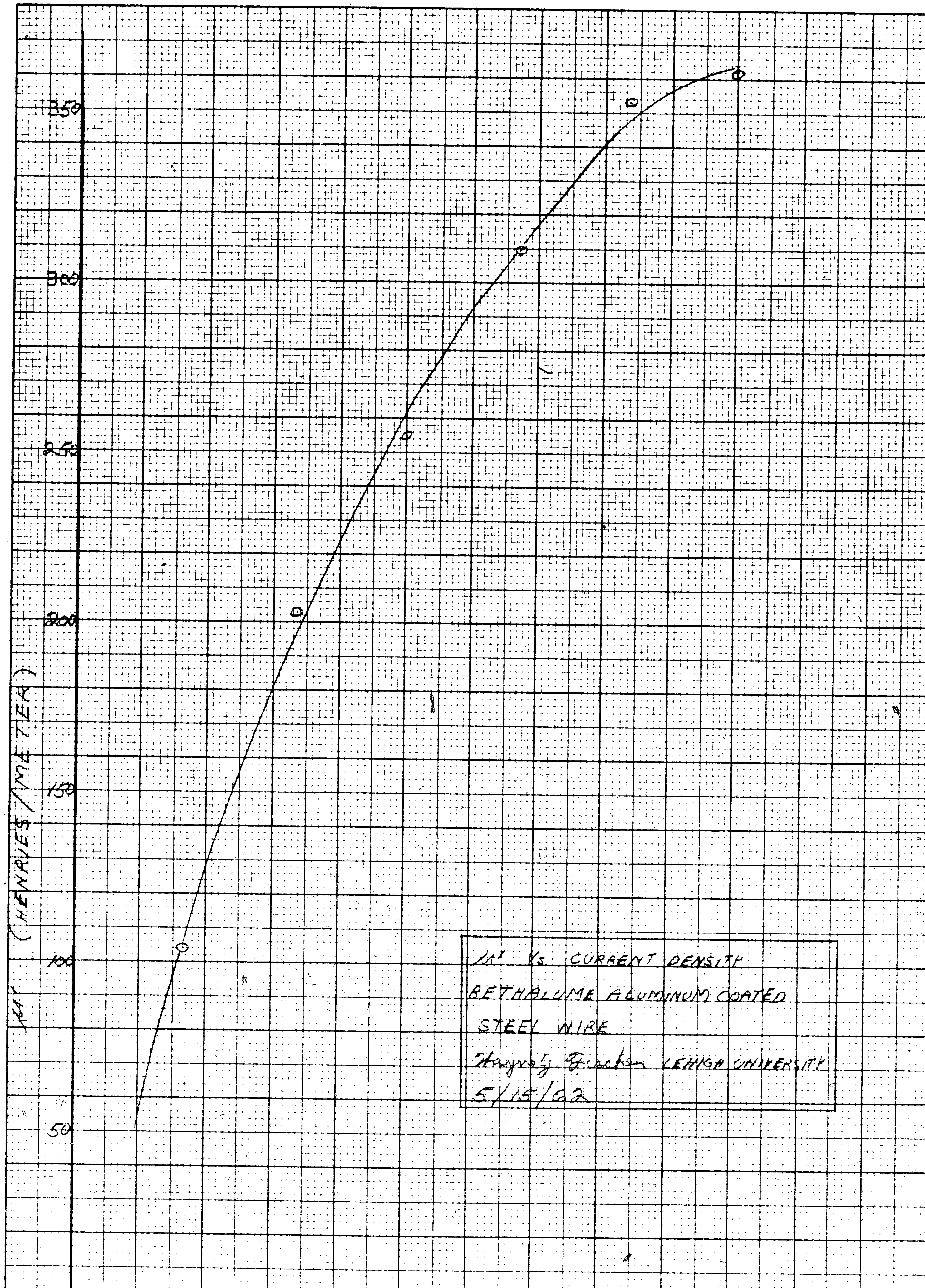

FIG. 5 μ' VS. CURRENT DENSITY

IV COMPARISON OF MEASURED AND CALCULATED VALUES

All calculations were based on a wire temperature of 20°C. To find the value of impedance at equilibrium temperature, the resistance and the internal component of the inductive reactance is multiplied by $(1+\alpha \Delta T)$, where α is equal to 0.0031 per °F. The actual value of the temperature coefficient for inductive reactance was not a constant value over the range of current densities. The values of modified permeability, μ', were determined on the basis of the temperature coefficient for inductive reactance being the same as that of the resistance in order to simplify the calculations.

The spacing component of the inductive reactance must then be added to the above value of impedance. The values of temperature rise as a function of current are given in Fig. 6.

In Fig. 7, 8, and 9, a comparison of the measured and calculated values is given.

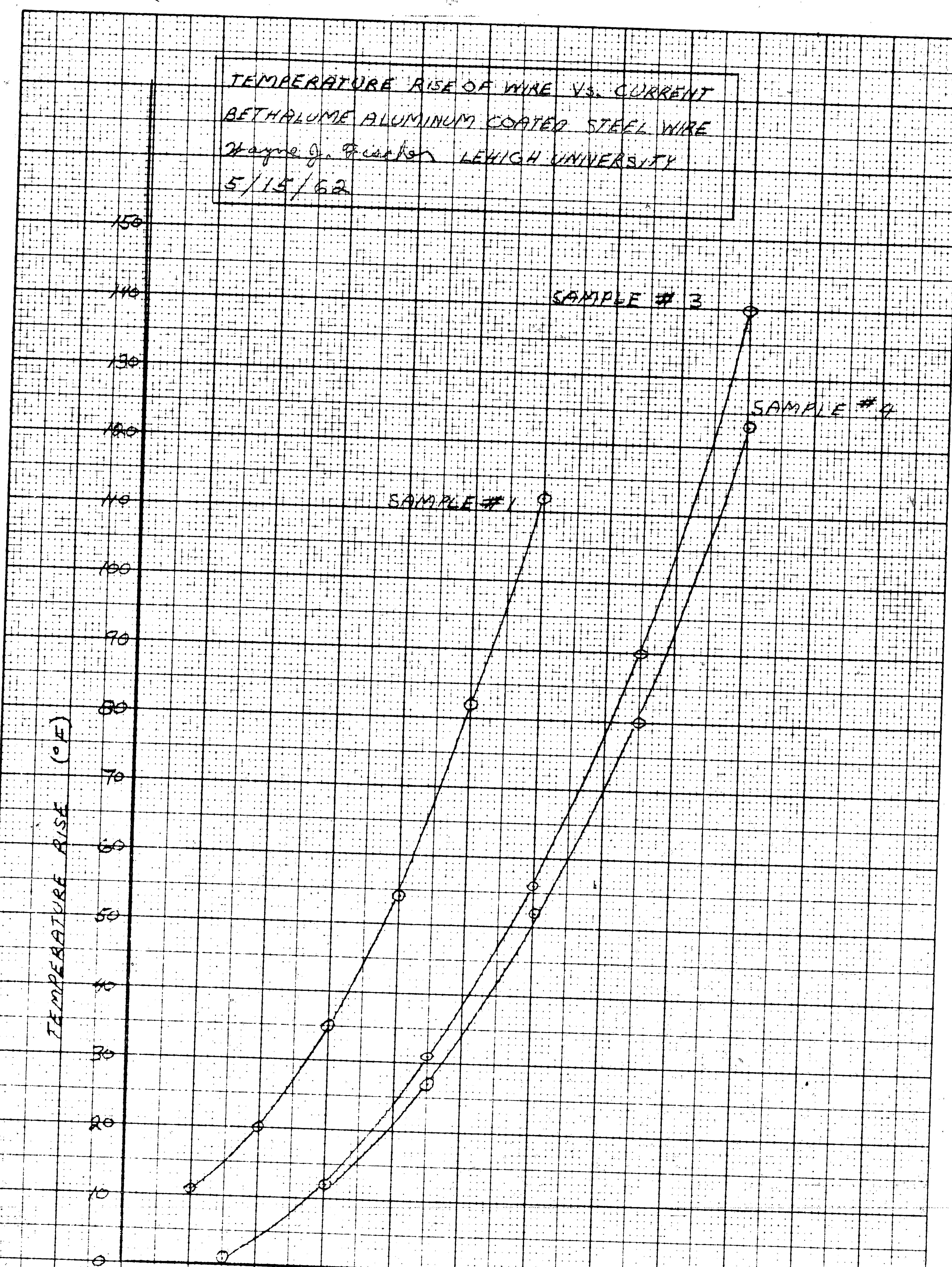

FIG. 6 TEMPERATURE RISE OF CONDUCTOR VS. CURRENT

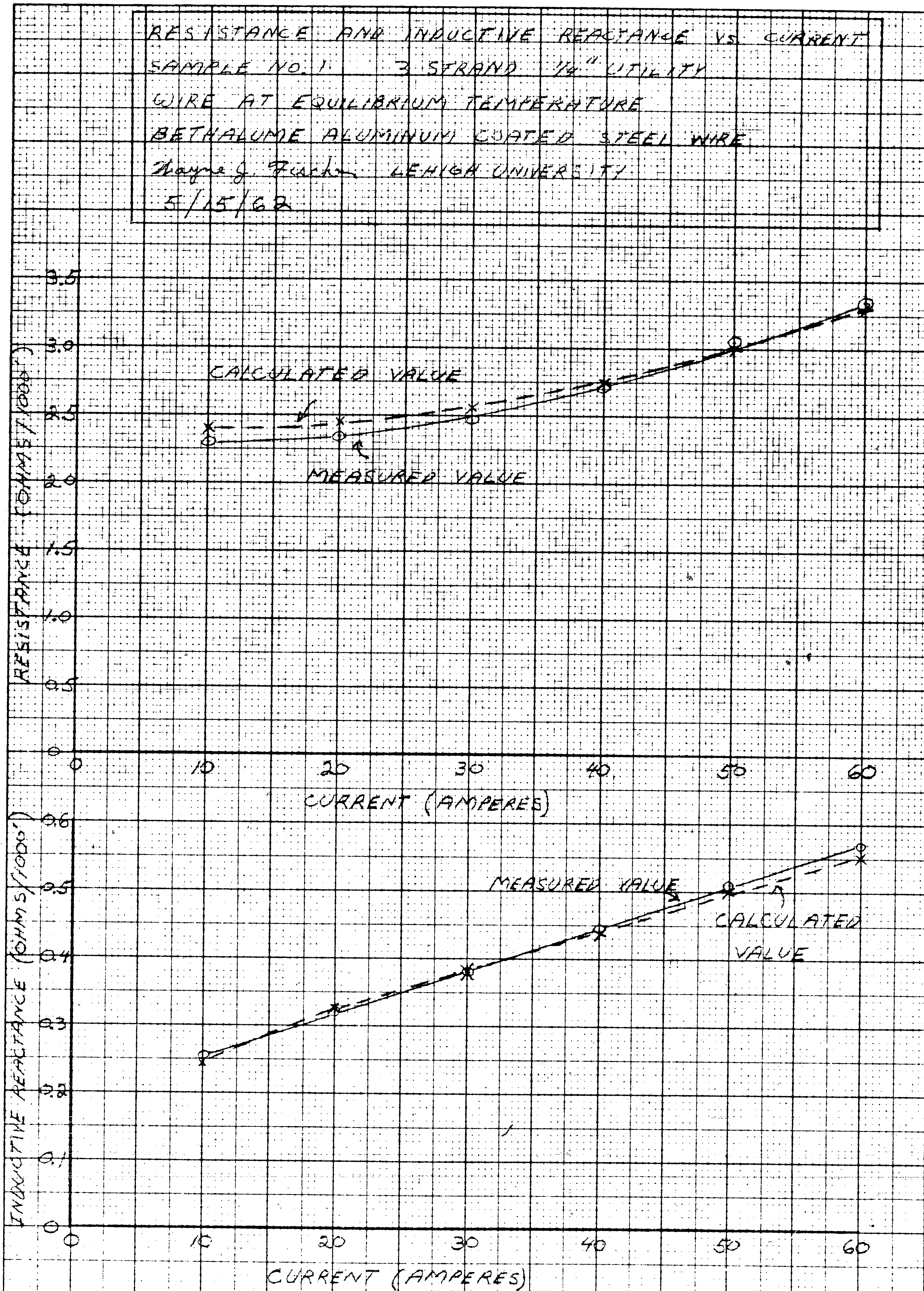

FIG. 7 RESISTANCE AND INDUCTIVE REACTANCE VS. CURRENT

RESISTANCE AND INDUCTIVE REACTANCE Vs CURRENT
SAMPLE NO. 3 3 STRAND 3/8" UTILITY
WIRE AT EQUILIBRIUM TEMPERATURE
BETHALUME ALUMINUM COATED STEEL WIRE
Wayne J. Fischer LEHIGH UNIVERSITY
5/15/62

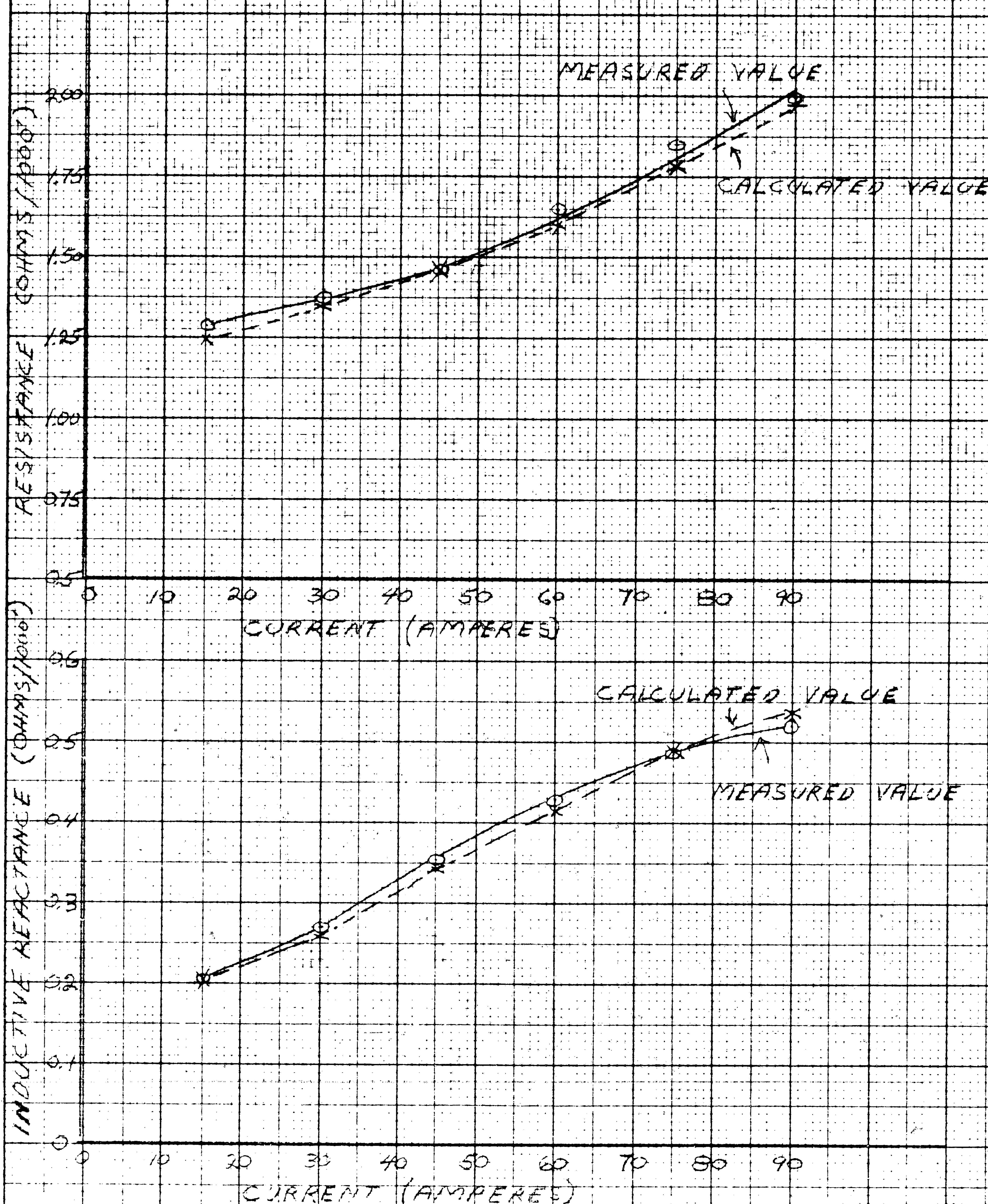

FIG. 8 RESISTANCE AND INDUCTIVE REACTANCE Vs. CURRENT

RESISTANCE AND INDUCTIVE REACTANCE VS CURRENT
SAMPLE NO. 4 3 STRAND 3/8" EXTRA HIGH STRENGTH
WIRE AT EQUILIBRIUM TEMPERATURE
BETHALUME ALUMINUM COATED STEEL WIRE
Wayne J. Fischer LEHIGH UNIVERSITY
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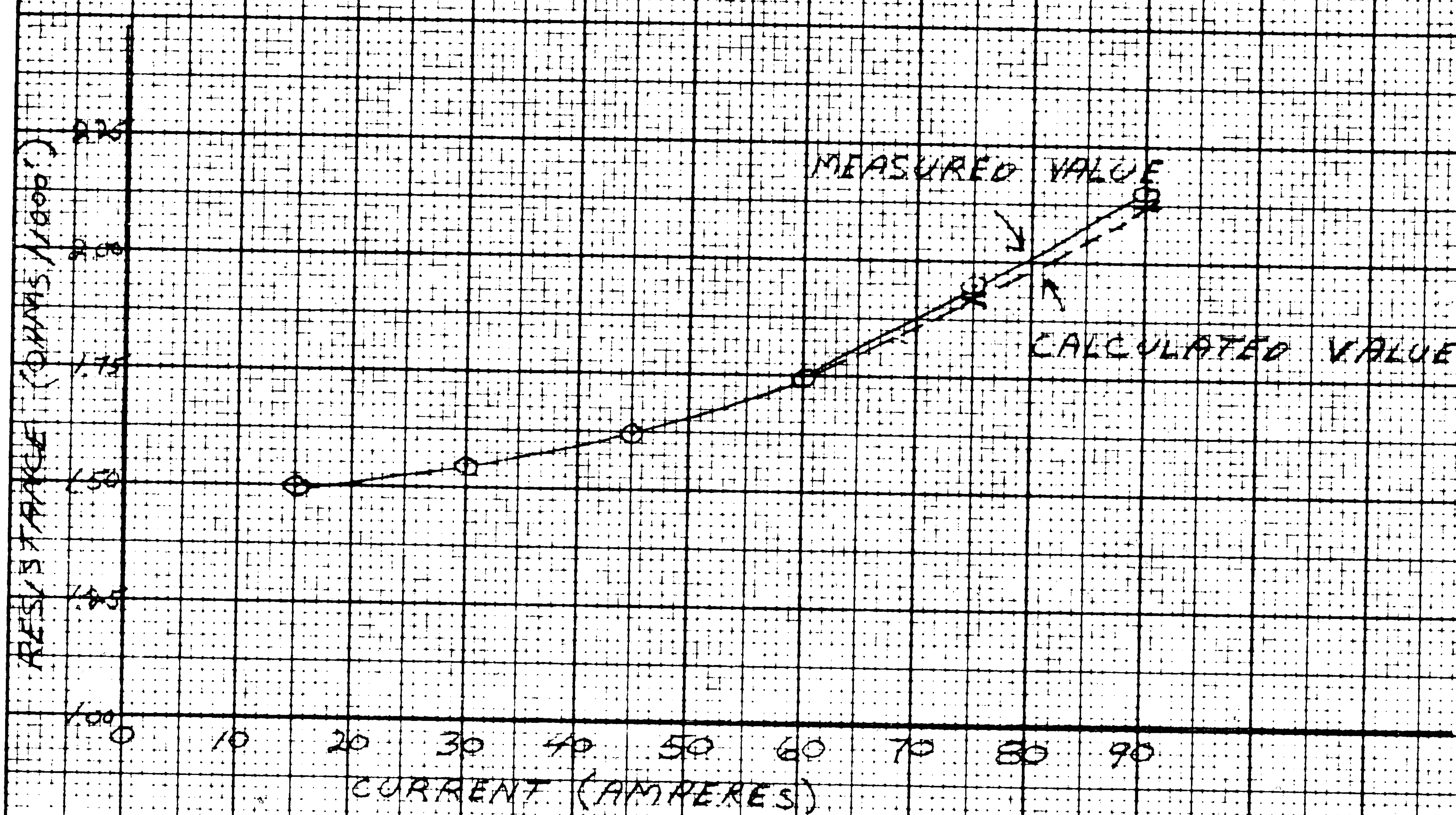

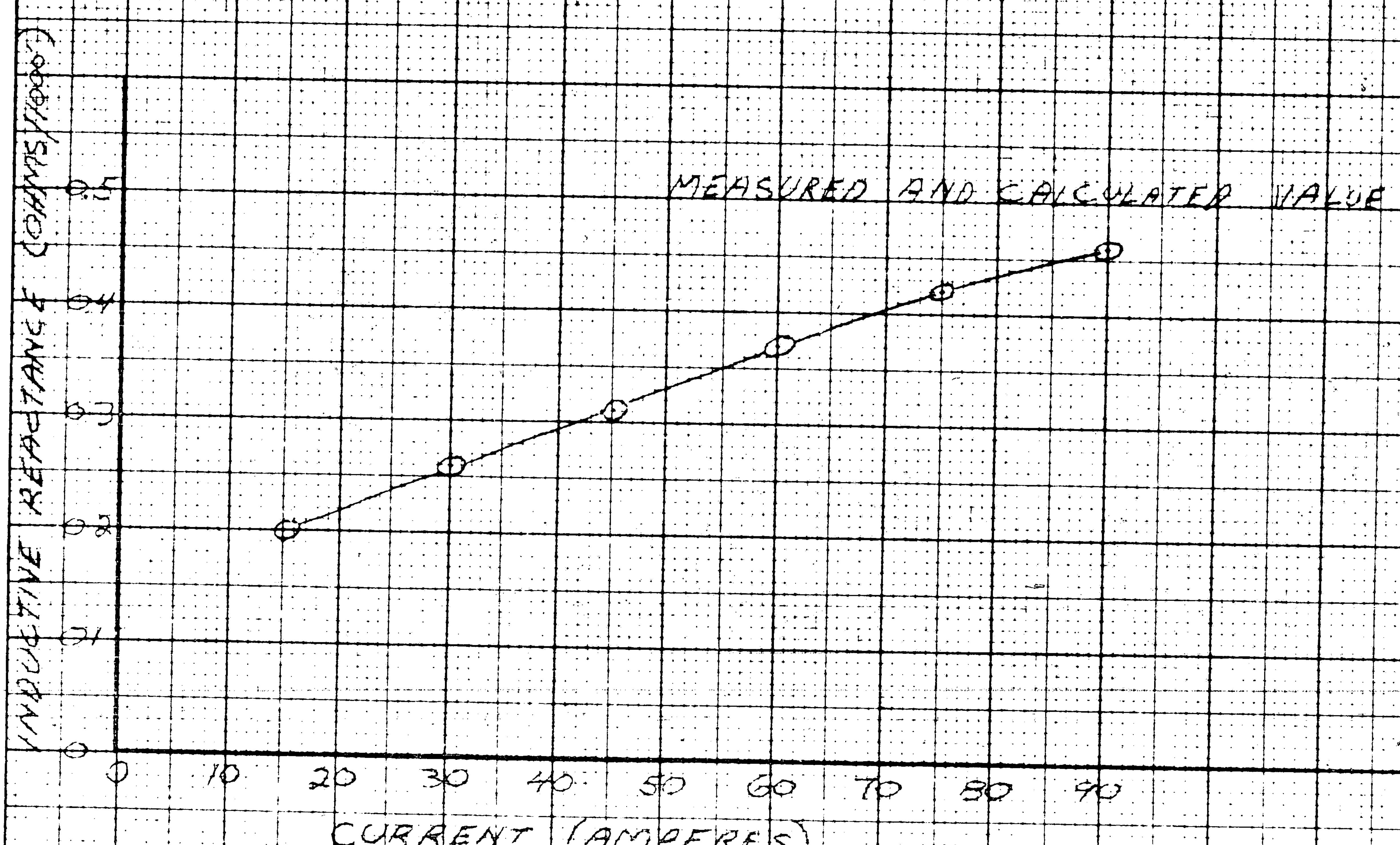

FIG. 9 RESISTANCE AND INDUCTIVE REACTANCE VS. CURRENT

V DISCUSSION

The method of calculation given above yields an exact solution to the problem of calculating the inductive reactance and resistance of a single strand of bimetallic conductor. For the case of three strands of conductor spiralled together, an equivalent value of permeability is found experimentally and yields satisfactory results. This equivalent value of permeability, μ', is found to be independent of the type of steel and the size of the conductors, and is dependent only on the configuration of the strands.

It is felt by the author that by using this value of μ', along with handbook values of resistivity for steel and aluminum, that inductive reactance and resistance can be calculated accurately for any three strand aluminum coated steel strand, within a reasonable range of the size and coating thickness of the samples tested.

The author was not able to perform a similar analysis on the seven strand samples tested. In this case, the additional skin effect involved, probably a sizable percentage of the current is carried by that portion of the steel nearest to the outside of the cable. If this is the case, the permeability depends on the type of steel, the size of the wire, and also the configuration of the strands. Only four seven strand samples were tested, but three different types of steel and three different sizes of conductor were involved. For the number of samples tested, the number of variables was too large to enable the author to draw any valid conclusions. No doubt the method presented in this thesis could be easily extended to seven strand conductors if more test data were available.

For the three strand samples tested, the calculated values were within three percent of the measured values throughout the range of current flow that was tested.

BIBLIOGRAPHY

1a. Hildebrand, F. B. Advanced Calculus for Engineers. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1949, p. 132-139

1b. ______. ditto, p. 152-165

2. Jahnke, E. and Emde, F. Tables of Functions with Formulae and Curves. New York: Dover Publications, 1945, p. 246-267

3. Jordan, E. C. Electromagnetic Waves and Radiating Systems. Englewood Cliffs, New Jersey: Prentice-Hall. Inc., 1950, p. 104-109

4. Kraus, J. D. Electromagnetics. New York: McGraw-Hill Book Company, Inc., 1953, p.128-131

5. Teare, B. R. Jr. and Webb, J. R. "Skin Effect in Bimetallic Conductors." Transactions of the American Institute of Electrical Engineers. Vol. 62, 1943, p. 297-302

6. Teno, J. and Fischer, W. J. "Report of Test Results of the Electric Characteristics of Bethalume Strand Cables." Lehigh University, 1962.

7. Woodruff, L. F. Principles of Electric Power Transmission. New York: John Wiley and Sons, Inc., 1938, p. 54-61

BIOGRAPHY

The author was born November 19, 1939, in Pittsburgh, Pennsylvania, the son of Helen and Frank Fischer. He attended Shaler High School in Glenshaw, Pennsylvania from 1952 to 1957.

From 1957 to 1961 he studied in the field of Electrical Engineering at Lehigh University, Bethlehem, Pennsylvania, where he received a Bachelor of Science degree in 1961. Since 1961 he has been engaged in graduate studies in Electrical Engineering at Lehigh University.

He has gained professional experience working for Westinghouse Electric Company, Atomic Equipment Division, during the summer vacations; working for the Lehigh University Institute of Research during the fall semester of the 1961 academic year; and working for the Department af Electrical Engineering, Lehigh University as a teaching assistant during the spring semester of the 1962 academic year.